Design of Digital Beamforming-based Automobile Collision Avoidance System

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***Abstract*—This paper describes the design of an automobile forward collision avoidance system based on digital beamforming techniques. The advantages of using digital beamforming over the existing system is stressed upon. In the proposed system, an adaptive array of antennas is used in the receiver side to form a pencil beam. The methodology of electronically controlling the direction of the pencil beam is described. Various methods of power spectrum estimation techniques are analyzed and compared both theoretically and through simulation in order to recommend the best approach for the proposed system. Linear prediction-based tracking algorithms are compared and the most accurate and efficient of them is identified. A DSP-based or ASIC-based implementation of this system can perform the entire collision avoidance functionality inside the host vehicle's on board computer.**

***Index Terms*—Adaptive array, digital beamforming, power spectrum estimation, linear prediction, Kalman filter**

I. Introduction

DRIVER errors are responsible for most of the car accidents. It has been identified that driver errors are a cause or severity-increasing factor in 93% of the accidents[1]. Out of these, nearly 28.4% of accidents are rear-end collisions[2]. Forward collision avoidance (FCA) system aims at avoiding or mitigating frontal collision of vehicles, including rear-end collision. There are two levels in which an FCA system can operate. At the first level, in which driver can still act to avoid accident, the FCA system alerts the driver either through a sound alarm or through a vibrating accelerator pedal. In the second level, the collision is avoided by a combination of automated braking and steering. The determination of which level, the FCA should resort to is based on a decision making system that considers relative velocity and relative distance.

Currently most of the FCA systems in the market are using global positioning system (GPS) to determine these two parameters. In a GPS based FCA system, the host vehicle is hooked with a GPS receiver that gets signal from GPS satellite. By this mean, the absolute position of the car is determined. Then the next task is to determine the nearest hurdle to the forward traversal of the host vehicle in its current velocity. Relative velocity and distance are deduced on the basis of absolute velocities of the vehicles and absolute positions. Although this system works well, it has some drawbacks. During night time or when the sky is heavily overcast, the GPS satellite may not be able to identify the hurdle or sometimes even the host vehicle. This may result in the system going out of order temporarily. In a worst case, the system may trigger a false alarm. GPS based system may not work when the host vehicle is not in an open environment, like parking lot or multi-tier highway. Other than the technical limitations, the owner of the host vehicle may be reluctant to install such a system, since the absolute position of the car is always available to a third party. It may be construed as an issue of privacy breach.

The solution for these problems is to build an FCA system in which the entire logic of collision avoidance can be embedded with in the host vehicle's computer. This paper aims to provide such a solution based on the digital beamforming technique. The digital beamforming technique can be used to measure both the relative velocity and position by suitably installing an array of sensors in the frontal portion of the car with digital signal processor hardware and software attached to it. Unlike the GPS based system both these parameters are measured directly reducing the complexity of the overall system. Once these are determined, decision making is then based on the probability density function for the relative position from the host vehicle to the nearest obstacle. The entire logic can be implemented in the host vehicle's computer directly or through an independent embedded system. Similar DSP algorithms based on beamforming system are used in seeker missiles where the motive is collision with the target, contrast to the road collision avoidance system.

The basic definition and advantages of digital beamforming and the architecture of the proposed collision avoidance system based on digital beamforming is given in Section II. The mathematical model and the design of the digital beamforming receiver system is described in Section III. Various methods of power spectrum estimation are analyzed in Section IV and different linear prediction algorithms are compared in Section V. Simulation results are presented in Section VI with comparison of different performance metrics. The work is summarized and conclusion is drawn in Section VII.

II. Digital beamforming

Beamforming is the combination of microwave signals from a set of small non-directional antennas to simulate a large directional antenna. Such a simulated antenna can be pointed to any direction with in a particular range electronically, without physically moving that antenna. In direction finding applications, beamforming can be used to steer an antenna to determine the direction of the signal, which is called the *angle of arrival* (AoA). The usage of an array of sensors instead of a single omni-directional dipole antenna is inspired from the way the eyes of cockroach works. Cockroach has multiple eyes in a plane and as a result it can see multiple independent image in distinct directions. Similar to that, an array of sensors can be used to generate a pencil beam constantly focused on the positional parameters of the target.

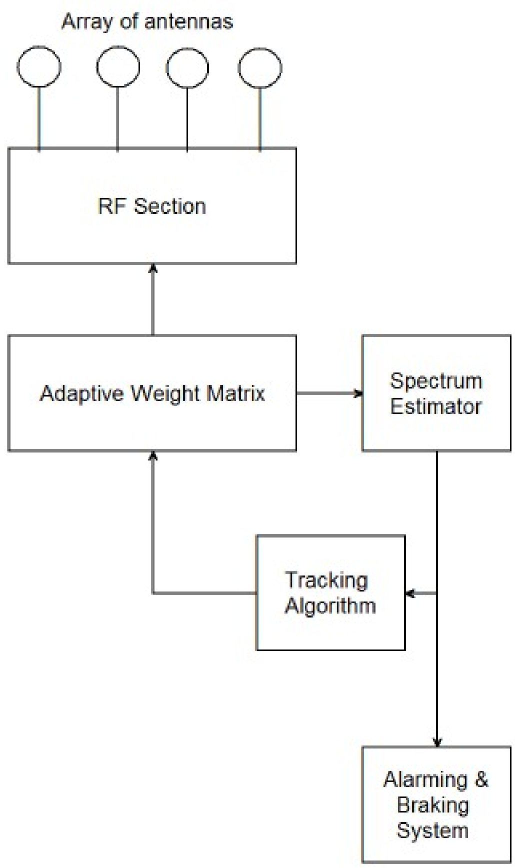

Figure 1. Block diagram of the proposed architecture

Digital Beamforming (DBF) is based on conversion of RF signal at each antenna elements into two streams of binary baseband signals representing inphase and quadrature phase channels. These two digital baseband signal can be used to recover amplitude and phase of the signal received at each element of the array. The process of digital beamforming implies weighing the individual antenna in the array by complex weight functions and then adding together to get the desired output.

The block diagram of the proposed collision avoidance system based on digital beamforming is shown in Figure 1. It consists of the array of beamforming antenna that forms the pencil beam in the microwave frequency in which the RF section is defined. The AoA of the beam is controlled by the adaptive weighting matrix. The weighted output is passed to a spectrum estimator. The spectrum estimator extracts the characteristics of the hurdle that includes its relative position and velocity and feeds it as input to the alarming and braking system of the host vehicle. These details also go to the tracking filter, which predicts the subsequent position and velocity of the hurdle. The output of the tracking filter goes as the feedback loop to the adaptive weighting matrix.

III. Digital Beamformer Receiver

A. Mathematical model

The transmitter system of the FCA system based on DBF can contain just a single transmitter antenna operating in microwave frequency. The receiver on the other hand, set up at the front portion of the car must be an array of sensors. An array of sensors can be arranged in any pattern in the three dimensional space. If a plane wave signal $f\left(t,p\right)$ arriving at an angle incidents on the array of sensors, the signal received by each sensor is going to be the same except for some time delay depending on the positioning of the sensors. The following vector can be used to describe the signal arriving at each sensor[3]:

$$f\left(t,p\right)=\begin{bmatrix} f_{p_0}\left(t\right) \\ f_{p_1}\left(t\right) \\ \vdots \\ f_{p_{N-1}}\left(t\right) \end{bmatrix}=\begin{bmatrix} f\left(t-\tau_0\right) \\ f\left(t-\tau_1\right) \\ \vdots \\ f\left(t-\tau_{N-1}\right) \end{bmatrix} \tag{1}$$

where N is the number of elements in the array and τ_i represents the time delay associated with the position of the i^{th} element of the array.

The design of the DBF receiver can be divided into four main stages namely Antenna design and RF modulation stage, digital down conversion stage, complex weight multiplication stage, and summation stage. Out of these, the first stage i.e. antenna design and RF modulation technically does not fall into the digital beamformer. So that part has not been considered in detail for analysis in this paper. However, the design of the antenna, RF reception, choice of the microwave frequency and choice of modulation technique are also essential in the implementation of a forward collision avoidance system.

The plane microwave frequency waveform that incidents on the array of sensors is given by the general equation (assuming the channel is completely noiseless):

$$f\left(t,p_k\right)\approx x\left(t\right)\cos\left(\Omega\left(t-\tau_k\right)\right),\ k=0,\,\ldots,\,N-1$$

$$f\left(t,p_k\right)=x\left(t\right)\cos\left(\Omega t-\theta_k\right)$$

,where $\theta_k=\Omega.\tau_k$, the phase change due to the time-delay with respect to the position of the k^{th} sensor, $x\left(t\right)$ is the transmitted RF wave with microwave frequency Ω. This RF signal is modulated to an intermediate frequency $\left(\omega\right)$ and passed through an ADC to obtain its digital representation given by:

$$g_k\left(n\right)=x\left(n\right)\cos\left(\omega n-\theta_k\right) \tag{2}$$

This digital signal is now passed into a digital down converter, which splits this signal into its inphase component and quadrature phase component. That can be achieved by multiplying this signal with a digital sinusoidal signal to get the inphase component and with a 90^o phase-shifted version of the same sinusoid to get the quadrature phase component. Usually the digital local oscillator that generates this sinusoidal signal will operate in the same frequency as the intermediate frequency ω. The inphase and quadrature phase signals are represented in the form[3]:

$$i'_k\left(n\right)=g_k\left(n\right).\cos\left(\omega n\right)$$

$$q'_k(n) = g_k(n) \,.\, \sin(\omega n)$$

Substituting the value of $g_k(n)$ from (2), in these equations, we will get:

$$i'_k(n) = \frac{x(n)}{2}\left(\cos 2\omega n + \cos\theta_k\right)$$
$$q'_k(n) = \frac{x(n)}{2}\left(\sin 2\omega n + \sin\theta_k\right) \quad (3)$$

The inphase and quadrature phase components are sums of a high frequency signal at twice the intermediate frequency and a low frequency signal at a frequency equal to phase of the signal representing the time-delay. Our object of interest out of these two is the low frequency signal carrying the phase information. So as the last step of the down converter, we subject the inphase and quadrature phase signals to a low pass filter. The result obtained can be given as:

$$i_k(n) = x(n)\cos\theta_k$$
$$q_k(n) = x(n)\sin\theta_k \quad (4)$$

The next stage of the DBF system is the scaling system. Here the inphase and quadrature phase components derived in (4) are multiplied by complex weights w_k^* associated with each antenna. To analyze this, let us consider the baseband signal $b_k(n)$ represented as:

$$b_k(n) = i_k(n) - jq_k(n)$$

Applying the values of $i_k(n)$ and $q_k(n)$ from (4),

$$b_k(n) = x(n)\left(\cos\theta_k - j\sin\theta_k\right)$$

Applying Euler's theorem,

$$b_k(n) = x(n)\,.e^{-j\theta_k} \quad (5)$$

It can be inferred from (5) that $b_k(n)$ is basically the transmitted signal $x(n)$ itself scaled by the complex constant associated with the phase of the received signal. Evidently if this signal is multiplied by the complex weighing factor $w_k^* = e^{j\theta_k}$, then the scaled value is given by:

$$\begin{aligned} y_k(n) &= w_k^*.x(n)\,.e^{-j\theta_k} \\ &= e^{j\theta_k}.x(n)\,.e^{-j\theta_k} \\ &= x(n) \end{aligned}$$

Thus we can conclude that by adjusting the value of the complex weights w_k^*, we can tune the digital beamforming receiver in any desired direction or for signals with any AoA, without physically gyrating the array itself. The last stage of the DBF receiver sums up the processed signal from all the sensors in the array and scales down the sum by a factor N to recover $x(n)$. One more observation that can be made out of the above mathematical derivation is its verisimilitude with the digital filter design. That is why the digital beamforming technique is otherwise called spatial filtering. Just as in the digital filter which selectively allows only a particular band of frequency, the digital beamformer selectively allows only signals from a particular direction. This can be observed from the fact that for any other signal from a different AoA resulting in some phase $\theta \neq \theta_k$, $y_k(n)$ won't be equal to $x(n)$ and so that signal would be rejected. The analysis and simulations in this paper focuses only on designing the system with single main lobe in the angle θ_k. However mathematical equations and techniques are available to extend the same analysis for multiple main lobes pointing to different AoAs[4].

B. *Receiver Design Considerations*

The design and practical implementation of a digital beamforming receiver is based on the mathematical model derived in the previous subsection, taking into account the physical limitations of the real components. Before considering the receiver design, the transmitter design is assumed to contain a single multi-directional dipole antenna operating in microwave frequency, transmitting the modulated form of the Barker sequence. The first component of the DBF receiver is the array of sensors and the RF section. As explained earlier, the RF design is beyond the scope of this paper. For the array of sensors to be operating with maximum efficiency, the distance between adjacent array elements must be $\frac{\lambda}{2}$, where λ is the wavelength of the transmitted wave. The output of this stage is the intermediate frequency signal. The digital representation of this signal is obtained by passing this signal through an ADC. The actual technique in which the A-to-D conversion is achieved is irrelevant. But the bit resolution of the ADC has to be high enough, as it is inversely proportional to the quantization error. Since each antenna of the sensor is connected to independent ADC, it is essential to use a single clock in the circuit. The synchronization is important since the weights are dynamically determined by the phase assuming that it is purely due to the AoA.

Once the digital representation is obtained from ADC, the rest of the logic can be implemented using a digital signal processor or an ASIC system. The digital signals are fed into a digital down converter logic. The digital down converter splits the digital signal into its inphase and quadrature phase component by multiplying it with phased sinusoids. The digital sinusoidal signal oscillator can be obtained through direct digital synthesis (DDS). The direct digital synthesis involves generation of sinusoidal signal by means of look-up table reference for amplitude values[5]. The generated signal must be synchronous with the input signal in terms of frequency and phase. Once the inphase and quadrature phase signals are obtained, they have to be filtered to remove the IF component and its harmonics. This can be obtained through a comb filter operating at a stopband frequency twice that of ω and filter gain of 2 to counter the division by 2 that happens during down conversion as shown in (3).

IV. Spectral Estimation

A. *Estimation Problem*

When the DBF receiver is implemented, it can be tuned in any direction by supplying appropriate weight functions. Until the exact direction of the hurdle is determined, the weights are adjusted periodically in such a way that the main lobe of the beam sweeps across the frame of reference. The basic problem considered in this section is the estimation of power density spectrum of the received signal from observing it over a finite time interval. The length of the received data sequence is a major limitation on the quality of the power spectrum estimate. The length of the record is determined by the rapidity with which the weights are changed in the receiver. An estimation scheme has to be chosen that satisfies the goal of selecting as short a data sequence as possible and still precisely deduce the spectral characteristics of different signal components in data sequence.

B. Non-parametric method for Estimation

Non-parametric method for the power spectrum estimation are a class of algorithms that make no assumption about the data sequence. These algorithms operate directly on the periodogram of the signal given by:

$$P_{xx}\left(f\right) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x\left(n\right)e^{-j2\pi fn}\right|^{2} = \frac{1}{N}\left|X\left(f\right)\right|^{2} \tag{6}$$

where $x(n)$ is the sample sequence that comes out of the DBF receiver and $X(f)$ is its Fourier transform. Periodograms can be used to detect and measure the "hidden periodicities" in the data and thus determine the frequency[6]. The periodogram can be computed by use of the DFT which in turn is efficiently computed by a FFT algorithm. When only a few points of the periodogram are needed, the Goertzel algorithm may provide a very efficient computation. The periodogram is not a consistent estimate of the true power density spectrum, because of the problems due to leakage and frequency resolution[7]. Since the estimated spectrum is completely based on the length of the data sequence, the frequency resolution of these methods is the equal to the spectral width of the rectangular window of length N which is approximately at $-3dB$ points. Improvement can be made to the periodogram based power spectrum estimation by applying different windowing techniques that are used in FIR filters. The most common of them are Bartlett method, Welch method and Blackmann method. Applying these windows to the periodogram for spectral estimation increases the computation complexity, but yields a better result.

The advantage of the non-parametric spectral estimation method are their relative simplicity and ease of computation using several FFT algorithms and windowing techniques. But this method requires the availability of long data records in order to obtain the necessary frequency resolution required by a DBF receiver with 16 sensors that can be employed in a Frontal Collision Avoidance system, especially when different frequencies used in this application are relatively closer. Furthermore the effectiveness of this method is affected by spectral leakage effects due to windowing, that is inherent in finite length records. Usage of different windows may improve frequency resolution but only at the cost of increase in leakage. The DBF receiver operates on the signal reflected by the hurdle, which may be another moving vehicle or any stationary object. Because of the characteristics of the object and the additive white Gaussian noise, the reflected signal may be weak. The spectral leakage masks weak signals that are present in the data. The basic limitation of the non-parametric method is the assumption that the power spectrum is periodic with a period N. Parametric methods for Estimation overcome these limitations by extrapolating the data beyond the observed interval.

C. Parametric methods of Estimation

The parametric methods are considered suitable for applications like FCA system where short data records are available and the channel is noisy. The parametric methods are based on modeling the data sequence $x(n)$ as the output of a linear system characterized by a rational system function of the form

$$H\left(z\right) = \frac{B\left(z\right)}{A\left(z\right)} = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1+\sum_{k=1}^{p} a_k z^{-k}}$$

The corresponding difference equation is

$$x\left(n\right) = -\sum_{k=1}^{p} a_k x\left(n-k\right) + \sum_{k=0}^{q} b_k w\left(n-k\right)$$

where $w(n)$ is the input sequence and $x(n)$ is the output sequence. If the same equation is extended for power density spectrum as

$$\Gamma_{xx}\left(f\right) = \left|H(f)\right|^{2}\Gamma_{ww}(f)$$

This model is called an *autoregressive-moving average* (ARME) *process* of order (p,q). In this process, if the value of q is zero, then $x(n)$ is called an *autoregressive* (AR) *process* of order p. The AR model is by far most widely used in spectrum estimation. The reasons are an increased frequency resolution and very simple linear equation for AR parameters. One of the parametric method for AR model is Yule-Walker method. In the Yule-Walker method, the autocorrelation function is estimated from the data sequence to solve the AR model parameters. In this method, it is desirable to use the biased form of autocorrelation estimate[7],

$$r_{xx}(m) = \frac{1}{N}\sum_{n=0}^{N-m-1} x^{*}(n).x(n+m)$$

to ensure that the autocorrelation matrix is positive semidefinite which results in a stable AR model. Applying the Levinson-Durbin algorithm[8], [9], the power spectrum estimate is given by

$$P_{xx}^{YW}(f) = \frac{\hat{\sigma}_{wp}^{2}}{\left|1+\sum_{k=1}^{p}\hat{a}_p(k)e^{-j2\pi fk}\right|^{2}} \tag{7}$$

where $\hat{a}_p(k)$ are estimates of the AR process obtained from Levinson-Durbin algorithm and

$$\hat{\sigma}_{wp}^{2} = r_{xx}(0)\prod_{k=1}^{p}\left[1-\left|\hat{a}_k(k)\right|^{2}\right]$$

is p^{th} order predictor estimate for *minimum mean-square error* (MMSE).

Yule-Walker method is computationally very efficient and yields a stable AR model. The efficiency of the entire algorithm depends on the implementation of Levinson-Durbin algorithm. In a single processor sequential system, the efficiency is $O(n^2)$. On the other hand, if the processing is performed on a parallel processing system, many of the mutually independent multiplications and additions can be carried out concurrently. If the number of parallel cores is equal to the number of poles in the AR model, the efficiency improves to $O(n \log n)$. In practical scenario, when the input signal is contaminated with additive white Gaussian noise then an extended Levinson-Durbin algorithm can be implemented[10]. With the advent of multi-core digital signal processors and application specific integrated circuits (ASIC), Yule-Walker algorithm can be performed efficiently to determine the power density spectrum. However the frequency resolution of Yule-Walker method deteriorates severely with lesser number of AR poles, even at very good *signal-to-noise ratio* (SNR).

For DBF system operating in frequencies very close to each other, either the number of poles has to be increased or unconstrained least-squares method of AR model has to be used. In this method, the constraint arising from Levinson-Durbin is not imposed for deriving the AR parameters. The unconstrained minimization of error ε_p with respect to the prediction coefficients yields the set of linear equations

$$\sum_{k=1}^{p} a_p(k).r_{xx}(l,k) = -r_{xx}(l,0) \qquad l = 1, 2, \ldots, p$$

where the autocorrelation function $r_{xx}(l,k)$ is

$$r_{xx}(l,k) = \sum_{n=p}^{N-1} [x(n-k).x^*(n-l) + x(n-p+l).x^*(n-p+k)]$$

The resulting residual least square error is

$$\varepsilon_p^{LS} = r_{xx}(0,0) + \sum_{k=1}^{p} \hat{a}_p(k) r_{xx}(0,k)$$

Hence the unconstrained least-squares power spectrum estimate is given by[11]:

$$P_{xx}^{LS}(f) = \frac{\varepsilon_p^{LS}}{\left|1 + \sum_{k=1}^{p} \hat{a}_p(k) e^{-j2\pi f k}\right|^2} \tag{8}$$

The unconstrained least-squares method gives excellent frequency resolution and very good performance even in low SNR compared to other methods. The correlation matrix is not toeplitz and so Levinson-Durbin algorithm cannot be applied in this method. Instead of that, Marple's algorithm can be applied with computational complexity proportional to $O(n^2)$[12]. Although not as efficient as Yule-Walker method, the trade-off is well justified in terms of its performance. With the unconstrained least-squares method, there is no guarantee that the estimated AR parameters would yield a stable AR model. But in spectral estimation, since we would not be applying the constant filter for a long time, this is not considered as a problem.

One of the important aspect of using the AR model is the selection of the order p. If the order is too low, highly smoothed spectrum is obtained. Such a model would not yield a good frequency resolution, especially in low SNR conditions. A low order AR model can be used in a closed, noise-free environment with operating frequencies are not aligned closer. In an FCA system, the array of sensors that is fitted on the frontal side of the vehicle, operates on the basis of the signal reflected by the hurdles on the road. The reflected signal is usually weak and high SNR cannot be expected. On the other hand, if the order of the AR model is too high, it introduces spurious peaks in the spectrum. One of the information criterion to select the value of p is based on the order that *minimizes the description length* (MDL)[13], where MDL is defined as

$$MDL(p) = N \ln \hat{\sigma}_{wp}^2 + p \ln N$$

An alternative criterion is called *criterion autoregressive transfer* (CAT) function[14] and is defined as

$$CAT(p) = \left(\frac{1}{N} \sum_{k=1}^{p} \frac{1}{\overline{\sigma}_{wk}^2} \right) - \frac{1}{\hat{\sigma}_{wp}^2}$$

where

$$\overline{\sigma}_{wk}^2 = \frac{N}{N-k} \hat{\sigma}_{wk}^2$$

The order p is selected to minimize $CAT(p)$. Despite having these criteria, it is apparent that in the absence of any prior information regarding the physical process, different model orders must be tried and the results must be used to determine the optimum order.

D. Eigen Analysis Algorithms for Estimation

An AR model explained in the previous section can act as a ARMA model in the presence of additive white noise. For such a noise-corrupted signals, eigen decomposition of the correlation matrix can be used to determine the frequency components. The underlying assumption for eigen-based estimation is that the signal is corrupted by noise, which is usually the case in a frontal collision avoidance system.

The *multiple signal classification* (MUSIC) is one of the noise subspace frequency estimator that considers the "weighted" spectral estimate[15], [16]. The MUSIC sinusoidal frequency estimator is a special case in which the weights are unity. The power density spectrum of MUSIC is given as

$$P_{MUSIC}(f) = \left(\sum_{k=p+1}^{M} \left| \mathbf{s}^H(f) v_k \right|^2 \right)^{-1} \tag{9}$$

where M is the number of autocorrelation lags, $\{v_k,\ k = p+1, \ldots, M\}$ are the eigen vectors in the noise subspace, p is the estimated number of sinusoids in the received signal, and $\mathbf{s}(f)$ is the complex sinusoidal vector given by

$$\mathbf{s}(f) = [1, e^{j2\pi f}, e^{j4\pi f}, \ldots, e^{j2\pi(M-1)f}]$$

The estimates of the sinusoidal frequencies are the peaks of $P_{MUSIC}(f)$. Once the sinusoidal frequencies are estimated, the power of each of the sinusoid can be calculated.

The MUSIC algorithm also provides the number of sinusoid components in the signal. Suppose there are p sinusoids, the eigen values associated with the signal subspace are $\left\{\lambda_i + \sigma_w^2,\ i = 1, 2, \ldots, p\right\}$ while the remaining $(M-p)$ eigen values are all equal to σ_w^2 that is the noise variance. If the eigen values of the sample autocorrelation matrix are ranked so that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$, where $M > p$, the number of sinusoids in the signal subspace is estimated by evaluating the minimum value of $MDL(p)$, given as[17]

$$MDL(p) = -\log\left[\frac{G(p)}{A(p)}\right]^N + E(p)$$

where

$$G(p) = \prod_{i=p+1}^{M} \lambda_i$$

$$A(p) = \left[\frac{1}{M-p}\sum_{i=p+1}^{M} \lambda_i\right]^{M-p}$$

$$E(p) = \frac{1}{2}p\,(2M-p)\log N$$

Computationally, MUSIC is costlier compared to nonparametric and parametric methods. But MUSIC exhibits excellent signal estimation performance even in very low SNR. Also the frequency resolution of MUSIC is far superior to the other methods. In MUSIC algorithm, the number of sinusoids in the received signal is estimated. In case of the FCA system, since the transmission is controlled by the system itself, the number of sinusoids in the reflected signal is precisely known. So this alleviates the problem of spurious peaks in high-order AR system.

E. Choice of the estimation algorithm

In case of a frontal collision avoidance system on the basis of digital beamforming, the system operates in an open environment. The channel is subject to huge noise, due to several factors that include the characteristic features of the object in front of the host vehicle, other communication devices and thermal noise of the nearby components. Since the channel quality is low, the nonparametric methods cannot be considered for our system. Other than the channel quality, non-parametric method expects a greater length of the data sequence to predict the sinusoids precisely. This implies that more time has to be spent at each AoA, which reduces the sweep speed and ability to identify the hurdle faster. This is clearly not acceptable, especially when the host vehicle is traveling at a high velocity.

On the other hand, the parametric methods show excellent performance even when the SNR is low. Parametric methods are not affected by spectral leakage, unlike non-parametric methods. But in parametric methods, care has to be taken to determine the order of the AR model, since non-optimal order can result in false alarm. Parametric methods are computationally costlier than nonparametric methods, which is based on computation of DFT on the basis of Goerzal algorithm. Out of the parametric methods, the frequency resolution of Yule-Walker method is poorer than even some windowed non-parametric methods. Yule-Walker method is preferable in our FCA system, if the multiple operating frequencies are sufficiently spaced, because of the computational efficiency of this method using Levinson-Durbin algorithm. Unconstrained least-squares method performs better than Yule-Walker method in low SNR and it does not have any frequency resolution problem, since it is not constrained by the toeplitz criterion. But this method is highly vulnerable to spurious peaks if the order is not chosen carefully. Also since Levinson-Durbin algorithm cannot be applied, it is computationally costlier.

For a generic digital beamforming system that is used in RADAR and seeker missiles, MUSIC algorithm is considered the best in identifying the reflected signal frequencies. It can work well even in a negative SNR and shows high frequency resolution. Unlike parametric methods, it is immune from problems due to spurious peaks. In an FCA system, since the transmitter is also a part of the system, we can accurately determine the number of sinusoids in the received signal. The only problem with the MUSIC algorithm is the requirement to calculate the eigen values and eigen vectors, which may prove computationally costlier. Besides, under high SNR which may occur when the host vehicle travels at a slow velocity in a remote garage, the peaks of the MUSIC algorithm may cause register overflow in the digital signal processor. This is not the problem of the algorithm and this is because of the limited number of bits with which the processor operates. So setting of the overflow flag has to verified in the logic that goes into the signal processor.

So the choice of the algorithm between parametric methods and MUSIC is really determined by the computational capacity of the FCA system. If a quad-core digital signal processor or an ASIC system is used as in the case of a seeker missile, MUSIC algorithm can be implemented. But if a single-core processor or an FPGA based system is used, then one of the parametric methods has to be resorted to, with a careful choice of the order of AR model and a compromise to be done on frequency spacing. The choice of the exact algorithm can be postponed until the simulated comparative results of these methods are discussed in VI-B on page 9.

V. Prediction and Tracking

A. Role of Prediction

In a forward collision avoidance system based on digital beamforming, the DBF receiver takes care of tuning the main lobe in a desired direction. At the beginning, the weights are adjusted in such a way that the beam is sweep in all directions in an attempt to gather the reflected signal wavefront in each direction. The angle in which the hurdle is located is determined by the power spectrum estimation methodologies described in the previous section. Once the AoA is determined, the relative position of the hurdle in a two dimensional plane is determined by the angle itself and the time taken for the microwave to hit the hurdle and gets reflected back to reach the receiver. The velocity of the vehicle is usually determined by the Doppler effect on frequency of the reflected wave, which can be obtained from the power spectrum estimation. If required, multiple main lobes can be formed to track multiple hurdles in a time sharing system. Once the relative position and relative velocity of the hurdle with respect to the host vehicle is determined, some prediction algorithms have to be applied to determine the course of the hurdle. The prediction algorithm chosen plays a major role in avoiding collision, especially when the hurdle detected is also a moving vehicle.

This helps us determine whether the vehicle in front is going to brake or swerve to left or right to change change lane or move ahead faster and trigger our alarming system accordingly. A forward collision avoidance system with prediction algorithm implemented is always more robust that the one without. Also with prediction algorithm incorporated the FCA system can be easily converted to a full-pledged adaptive cruise control system.

In an FCA system, the signal can be modeled as a sum of multiple sinusoid with additive white Gaussian noise. The adaptive filter that has to be designed for prediction has to suppress the undesired interference while preserving the characteristics of the original signal. The adaptive filter is constrained to be linear with an impulse response $h(n)$, designed in such a way that the output of that filter approximates some desired signal sequence $d(n)$. The input sequence to the filter is $x(n) = s(n) + w(n)$, where $s(n)$ is the actual signal which represents a sum of sinusoids and $w(n)$ is the additive noise. If this $x(n)$ goes through an optimum linear filter with characteristics $h(n)$, it produces $y(n)$ which is the approximation of the desired signal $d(n)$. The error in the approximation is given by $e(n) = d(n) - y(n)$. In this estimator system, if $d(n) = s(n + D)$, where $D > 0$, the linear estimation problem is referred to as signal prediction. The criterion selected for optimizing the filter response $h(n)$ is the minimization of mean-square error (MMSE).

B. Wiener Filter

The optimum linear filter in the sense of MMSE is called a *Wiener* filter. Suppose an FIR filter is constrained to an order M with coefficients $\{h_k, 0 \leq k \leq M-1\}$[18]. Hence the output $y(n)$ depends on the finite data record $x(n)$, $x(n-1)$, ..., $x(n-M+1)$,

$$y(n) = \sum_{k=0}^{M-1} h(k).x(n-k)$$

The mean square error between the desired output $d(n)$ and $y(n)$ is given by

$$\varepsilon_M = E\left|e(n)\right|^2 = E\left|d(n) - \sum_{k=0}^{M-1} h(k).x(n-k)\right|^2 \quad (10)$$

This equation (10) can be expanded as

$$\varepsilon_M = E[d(n)^2] - 2E\left[d(n).\sum_{k=0}^{M-1} h(k).x(n-k)\right]$$

$$+E\left[\left(\sum_{k=0}^{M-1} h(k).x(n-k)\right)^2\right]$$

This can be re-expressed as

$$\varepsilon_M = \sigma_D^2 - 2\sum_{k=0.}^{M-1} h(k)\gamma_{dx}(k) + \sum_{k=0}^{M-1}\sum_{m=0}^{M-1} h(k)h(m)\gamma_{xx}(m-k)$$

This is a quadratic function of the filter coefficients, the minimization of ε_M yields the set of linear equations

$$\sum_{k=0}^{M-1} h(k)\gamma_{xx}(m-k) = \gamma_{dx}(m)$$

This linear equation that specify the optimum filter is called *Wiener-Hopf* equation. The optimum value of $\{h(k),\, k = 0, 1, \ldots, M-1\}$ is given in matrix form by

$$\mathbf{h}_{opt} = \mathbf{\Gamma}_M^{-1}\boldsymbol{\gamma}_d \quad (11)$$

and the resulting MMSE is given by

$$MMSE_M = \sigma_d^2 - \boldsymbol{\gamma}_d^{*t}\mathbf{\Gamma}_M^{-1}\boldsymbol{\gamma}_d \quad (12)$$

The autocorrelation matrix to be inverted is toeplitz. The inversion can be done using Levinson-Durbin algorithm may be used to solve for the optimum filter coefficients. Alternatively the solution can also be obtained by means of Singular Value Decomposition (SVD) method[19], [20].

C. Discrete Kalman Filter

The Kalman filter[21], [22], [23] is a recursive solution to the discrete-data linear filtering problem. It addresses the general problem of trying to estimate the state $x \in \Re^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation:

$$x(k) = Ax(k-1) + Bu(k-1) + w(k-1) \quad (13)$$

with a measurement of $z \in \Re^m$ that is

$$z(k) = Hx(k) + v(k) \quad (14)$$

In these equations, the random variables $w(k)$ and $v(k)$ represent the additive white process noise and measurement noise respectively. They are mutually exclusive functions with normal probability distribution given by

$$p(w) \sim N(0, Q),$$

$$p(v) \sim N(0, R)$$

In practice the noise covariances Q and R might change with each measurement. For the *a priori* state estimate, $\hat{x}_k^{\sim}$ at each step k which is based on the process prior to the step k, the estimated error is given by

$$e_k^{\sim} = x_k - \hat{x}_k^{\sim}$$

The *a priori* estimate error covariance is given by

$$P_k^{\sim} = E\left[e_k^{\sim} e_k^{\sim T}\right]$$

Once the value of x_k is predicted by the predictor equation of the Kalman filter, an *a posteriori* estimate is made in an attempt to minimize the prediction error. This *a posteriori* state estimate is used to determine the next step. The *a posteriori* state estimate $\hat{x}_k$ is expressed as a linear combination of the $\hat{x}_k^{\sim}$ and a weighted difference between the actual measurement z_k and a measurement prediction. It is given by

$$\hat{x}_k = \hat{x}_k^{\sim} + K\left(z_k - H\hat{x}_k^{\sim}\right) \quad (15)$$

In Kalman filter terminology, the equation (13) is called *predictor* equation and (15) is called the *corrector* equation of the Kalman filter. The difference $\left(z_k - H\hat{x}_k^{\sim}\right)$ is called the measurement innovation of the Kalman filter and the matrix K is called the *Kalman gain* or *blending factor* that minimizes the *a posteriori* error covariance, which is given by

$$K = \frac{P_k^{\sim} H^T}{H P_k^{\sim} H^T + R} \tag{16}$$

Thus the Kalman filter during its operation first makes the *a priori* estimate on the basis of the predictor equation. Then the calculated *a priori* estimate is corrected by computing the *Kalman gain, K*. The final step is to obtain an *a posteriori* error covariance. At each time and measurement update pair, the process is repeated with previous *a posteriori* estimates used to project or predict the new *a priori* estimates.

The most appealing nature of the Kalman filter is this recursive nature. It makes the practical implementation much more feasible that the Wiener filter which is designed to operate on all data directly for each estimate. The Kalman filter instead recursively conditions the current estimate on all of the past measurements. Kalman filter not only predicts the next step, but also corrects the earlier prediction by taking the measurement error into consideration. In contrast the Wiener filter does not work on feedback mechanism. As a result the learning curve of the Kalman filter is steeper than that of the Wiener filter.

In the actual implementation of the Kalman filter in a DBF based FCA system, the measurement noise covariance R is usually measured prior to the operation of the filter. The determination of the process noise covariance Q is more difficult, since the operating environment of the FCA system is highly varying. Often the Kalman filter's performance becomes superior by tuning the filter parameters Q and R to a more realistic value. This can be performed off-line during system identification, by analyzing the most probable operating environment of the FCA system.

During the analysis of the operating environment, if the relation between the process and measurement coefficients are found to be non-linear then the Kalman filter may not be able to predict with high accuracy. In such situations, an extended Kalman filter (EKF) has to be resorted to[24]. EKF is a modified Kalman filter which linearizes the current mean and covariance. The complete equations of an EKF can be given only on the basis of the exact non-linear relation that exists in the environment. But the basic operation of the EKF is in the same predictor-corrector form of a linear discrete Kalman filter.

VI. Simulation

A. Beam pattern of DBF receiver

Theoretically, an array of sensor can be arranged in any pattern in a three dimensional space. The equation (1) gives the function of signal at each element of the array. The signal $f(t,p)$ generated in space can be considered a far-field wave, since the distance between the adjacent elements is negligibly low compared to the distance between the host car and the nearest hurdle. For a far-field wave, (1) can be rewritten as:

$$f(t,p) = \begin{bmatrix} f(t)\, e^{j\left(\Omega t - k^T p_0\right)} \\ f(t)\, e^{j\left(\Omega t - k^T p_1\right)} \\ \vdots \\ f(t)\, e^{j\left(\Omega t - k^T p_{N-1}\right)} \end{bmatrix}$$

where k is called wave number and Ω is the angular frequency of the operating microwave. The wavenumber k is the transformation parameter to convert Cartesian coordinates in which the array of sensors is represented to polar coordinates and p represents the position in cartisian coordinates. They are represented as follows:

$$k(\theta,\phi) = \frac{2\pi}{\lambda} \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}, \qquad p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \tag{17}$$

, where λ is the operational wavelength, θ is the planer angle and ϕ is the azimuth angle. To get the frequency characteristics of the received signal, we have to take Fourier transform to $f(t,p)$ which is:

$$F(\Omega,p) = \int f(t,p)\, e^{-j\Omega t} dt$$

Expanding the matrix,

$$F(\Omega,p) = \begin{bmatrix} \int f(t)\, e^{j\left(\Omega t - k^T p_0\right)} e^{-j\Omega t} dt \\ \int f(t)\, e^{j\left(\Omega t - k^T p_1\right)} e^{-j\Omega t} dt \\ \vdots \\ \int f(t)\, e^{j\left(\Omega t - k^T p_{N-1}\right)} e^{-j\Omega t} dt \end{bmatrix}$$

$$F(\Omega,p) = F(\Omega) \begin{bmatrix} e^{-jk^T p_0} \\ e^{-jk^T p_1} \\ \vdots \\ e^{-jk^T p_{N-1}} \end{bmatrix} = F(\Omega)\, v(k)$$

To determine the beam pattern of the receiver, we have to sum the array manifold vector $v(k)$ for each array element, scaled by the weights. Thus the beam pattern of the receiver antenna is given by the equation:

$$B(k) = \sum_{l=0}^{N-1} w_l^* v_l(k)$$

For a narrowband beamformer that can be used for our purpose to form a pencil beam, the weighting function is given by:

$$w_n^* = \frac{1}{N} e^{jk^T(\theta,\phi) p_n}$$

The equations of beam pattern and weight functions are applicable only for a generalized planer array where both the planer angle θ and azimuth angle ϕ are significant. But when we consider a linear array of sensors, which is commonly used in applications where there is no vertical movements to be tracked, the value of azimuth angle ϕ becomes zero. For constructing a frontal collision avoidance system, a linear array of sensors would suffice since the vertical movement of the host vehicle relative to the hurdle is fairly insignificant. If we apply the generalized equation (17) for a linear array, p_x and p_y would becomes zero and only p_z is significant. Under this assumption, the weight function becomes:

$$w_n^* = \frac{1}{N}\, e^{j\left(\frac{2\pi}{\lambda}\right)p_z \cos\theta}$$

For simulation purpose, let us assume that the DBF receiver has a linear array with 4 antennas with an inter-element spacing between individual antennas in the array must be $\frac{\lambda}{2}$ for optimum performance. For an AoA of 60^o, the beam pattern can be plotted in a polar graph as in Figure 2. In Figure 3, the beam pattern is plotted with 16 element linear array instead of 4 elements. From these figures, it can be noticed that the beam width of the main lobe is lower as the number of antennas in the linear array increases.

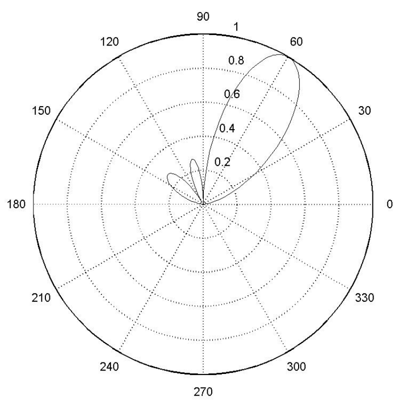

Figure 2. Beam pattern of linear array with 4 antennas for AoA of 60^o

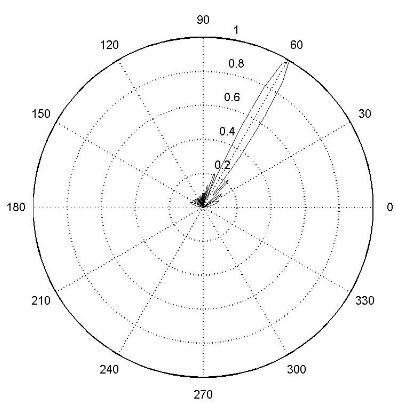

Figure 3. Beam pattern of linear array with 16 antennas for AoA of 60^o

B. Comparison of Estimation Algorithms

In this subsection, several spectral estimation algorithms are compared on the basis of their frequency resolution, bias, their robustness in the presence of additive white Gaussian noise. As already mentioned in IV-E on page 6, the performance of periodogram based methods would be subpar to be considered for implementation of a forward collision avoidance system. So the non-parametric methods are not considered for simulation. Both the parametric methods and MUSIC algorithm are considered for simulation and comparison.

The data model for this simulation consists of two sinusoids and additive white Gaussian noise. The two sinusoids are spaced Δf apart in the digital frequency domain ranging between $-\frac{1}{2}$ and $\frac{1}{2}$. The input data is represented in the form

$$x(n) = \cos 2\pi f n + \cos 2\pi (f + \Delta f)n + w(n)$$

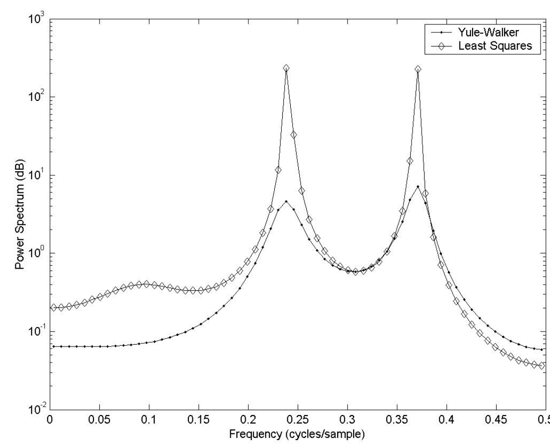

Figure 4. Comparison of AR spectrum estimation methods. AR(6), 20 pts, SNR = 20dB, Δf = 0.13

The initial comparison is between Yule-Walker method and the unconstrained least-squares method for $N = 20$ data points based on AR model of order 6, with an SNR = 20 dB and Δf = 0.13. It is illustrated in figure 4. It can be noted that both Yule-Walker method as well as least squares method yield a very good result. But at the same time, when the order of the AR model is reduced to 4, Yule-Walker method yields an extremely smooth spectral estimate with small peaks, which is illustrated in figure 5. This proves that by choosing the order properly and increasing the number of data points, Yule-Walker can be made to yield as good a result as least squares method. However, least squares method is clearly superior for short data lengths. Also when the value of Δf is reduced to 0.09 with AR(4), Yule-Walker method no longer identify the two frequency points distinctly, as illustrated in figure 6. On the other hand, even though least squares method smoothened as the value of Δf is reduced, it can still distinguish the two frequencies. This proves that least squares algorithm has a higher frequency resolution than Yule-Walker method.

Figure 7 illustrates the occurrence of spurious peaks in least squares algorithm in case of a higher order of AR model. For this illustration, the order chosen was 12. It can be noticed that least-squares is more vulnerable to these spurious peaks (4 peaks for 2 distinct frequencies) due to higher order than

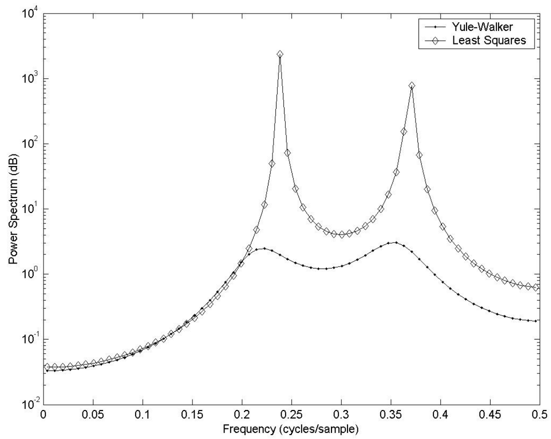

Figure 5. Comparison of AR spectrum estimation methods. AR(4), 20 pts, SNR = 20 dB, Δf = 0.13

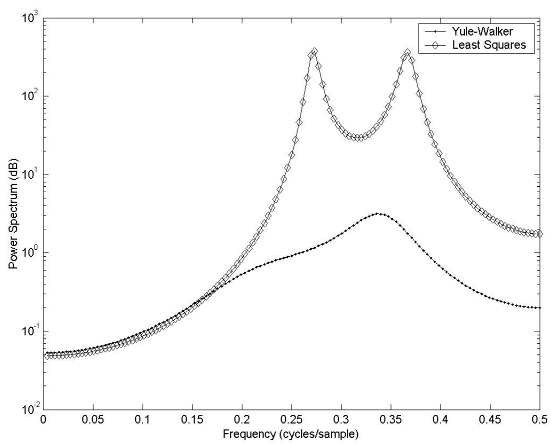

Figure 6. Comparison of AR spectrum estimation methods. AR(4), 20 pts, SNR = 20 dB, Δf = 0.09

the Yule-Walker method. These spurious peaks may result in false alarm in an FCA system and so completely undesirable. Thus even though the performance of the least squares method is superior to that of Yule-Walker method, unless the value of the order of AR model is chosen carefully, it would not serve our purpose in FCA. The theoretical guideline for choosing the order is to minimize the MDL function or CAT function. The simulation results suggest the value of the order of AR model to be chosen around $\frac{N}{3}$, for optimum performance of both the parametric methods.

Since it has been illustrated that the least squares method shows best performance among the parametric methods, it can be compared with MUSIC algorithm to determine which one is better. Theoretically, MUSIC algorithm shows excellent performance in a noisy environment as its power density spectrum directly depends on the noise-space eigen vectors. Also MUSIC algorithm does not have the concept of spurious peaks. Figure 8 illustrates the performance of least squares method of AR(6) and MUSIC algorithm at an SNR of 10 dB. Both the algorithms perform equally well at a 10 dB

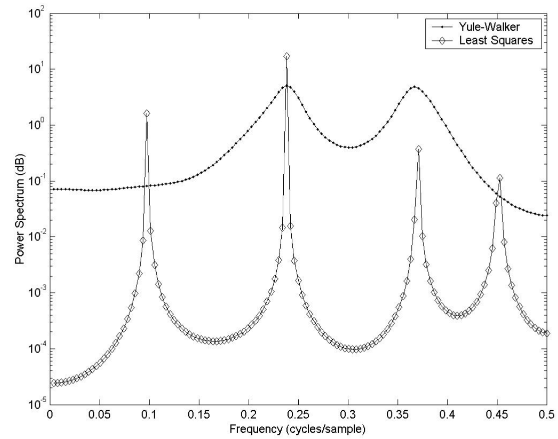

Figure 7. Comparison of AR spectrum estimation methods. AR(12), 20 pts, SNR = 20 dB, Δf = 0.13

SNR. So when the SNR of the FCA system is measured to be in the range of 10 dB, then it is a good idea to choose the least squares method, since it can be efficiently implemented using Marple's algorithm. The implementation of MUSIC algorithm is computationally costly since it involves eigen vectors calculation.

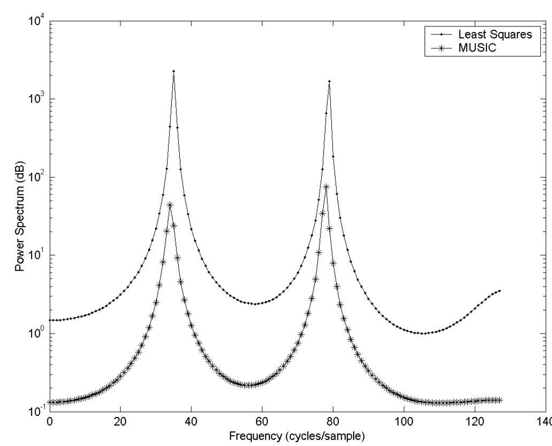

Figure 8. Comparison of least squares methods and MUSIC algorithm. AR(6), 20 pts, SNR = 10 dB, Δf = 0.13

Whenever the expected SNR is positive, least squares method can be chosen over MUSIC algorithm. When SNR becomes negative, the performance of MUSIC algorithm is much superior to that of least squares method. This is illustrated in Figure 9, which compares the power density spectrum of least squares method and MUSIC algorithm at -20 dB SNR, which signifies a noise that is 100 times more powerful than the signal. Because of this superior performance, MUSIC algorithm is preferred in military environment, as it can detect the signal even in the presence of high power jammer signals. A forward collision avoidance (FCA) system in front of a car may not require such a sophisticated algorithm at the expense of computation cost, since that much amount of additive noise

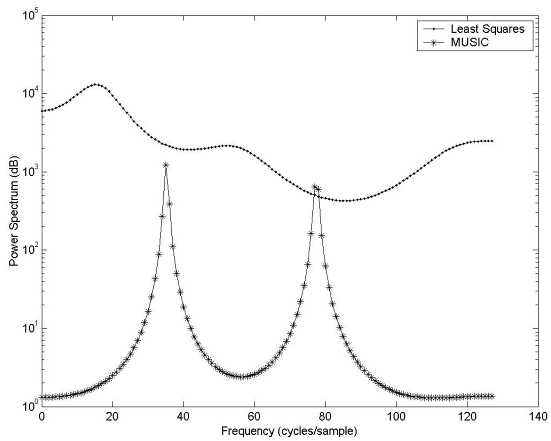

Figure 9. Comparison of least squares methods and MUSIC algorithm. AR(6), 20 pts, SNR = -20 dB, Δf = 0.13

need not have to expected in a domestic environment. So through these simulations, it can concluded that unconstrained least squares method with a carefully chosen order of the AR model and frequency gap is the most optimal implementation for the FCA system.

C. Comparison of Linear Prediction Algorithms

The two prediction algorithms that have been taken into account for tracking of the hurdle in an FCA system are Wiener filter and Kalman filter. The two different values that needs to be predicted in this system are the relative distance and relative velocity at the next instance with the values at the current instance and previous instances. The mean square error in prediction is the mean square difference between actual values measured at that instance and the predicted values at the previous instance.

The data model for this calculations is expressed as follows:

$$s = \begin{bmatrix} d \\ v \end{bmatrix} = \begin{bmatrix} d \\ \dot{d} \end{bmatrix} \tag{18}$$

where $\dot{d}$ is the differential of distance which is the velocity v. In a three dimensional space, distance and velocity are represented in the form in Cartesian coordinates (x, y, z). In an actual scenario of host vehicle traveling on a road, the relative vertical movement of the hurdle with respect to the host vehicle is negligible and irrelevant. This is the same reason why a linear array can be used in the FCA system and a planer array may not be required. Assume z-axis to represent the vertical direction which can be neglected. Thus the data model in (18) is deduced in Cartesian and polar coordinates as

$$s = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \equiv \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} \tag{19}$$

Since the main lobe of the beam pattern is represented in radial form, the same pattern could be followed here. The results of the predictive filters however are independent of which form is used.

For simulation, assume the relative distance between the host vehicle and the target vehicle is 30 meters at an angle of 60^o, which is moving from right lane to the lane in which the host vehicle is operating. The host vehicle is assumed to be traveling at 40 *meters-per-second* (mps) and the target vehicle at 50 mps. So the relative velocity of the target vehicle is +10 mps. The relative direction component of the target vehicle can be assumed as 3^o per second, i.e. the target vehicle is moving into the driving lane of the host vehicle at a rate of 3^o per second. Applying these values in the equation (19),

$$s = \begin{bmatrix} 30 \\ 60 \\ 10 \\ 3 \end{bmatrix}$$

The target vehicle is slowing down at a random rate, in such a way that the relative distance and relative velocity between the target vehicle and the host vehicle is reducing, while the relative direction is getting aligning to 90^o. This is a typical scenario in which the main lobe of the beam has to turn in such a way that the target vehicle is tracked without missing. The process noise power and measurement noise power are each assumed to be $\sigma_w^2 = 4$, both in distance and velocity fronts. Apart from these there is also a slight degree of randomness involved in the distance and velocity front. Thus the changing relative distance and velocity can be given as

$$s_l = \begin{bmatrix} 30 - l\sigma \\ 60 + 0.2l\sigma^2 \\ 10 - 2l\sigma^2 \\ 3 + 0.05l\sigma^2 \end{bmatrix} \qquad l = 1, 2, \ldots, L$$

, where σ^2 represents the randomness with unit variance (scaling is done in the equation) involved in the manual driving operation and L is the total number of iterations for which the tracking is done. In this described scenario, the performance of corrected value of Kalman filter and estimated value of Wiener filter are compared in Figure 10.

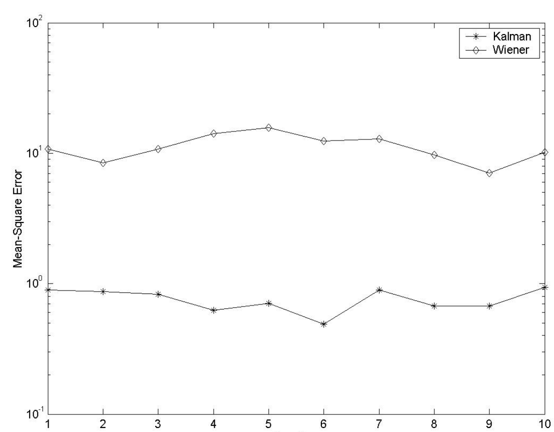

Figure 10. Comparison of Wiener filter and Kalman filter

It is evident from the comparison that the performance of Kalman filter is superior to the Wiener filter in the assumed scenario. The mean square error of the Kalman filter is nearly ten times lower than that of Wiener filter. The superior performance of the Kalman filter can be attributed to several factors including predictor-corrector system, consideration of measurement error also in the prediction and continuous learning nature. The prediction error of the Kalman filter alone may be greater than that of Wiener filter, but the corrector part of the Kalman filter moves the value closer to the actual. From the implementation point of view, Kalman filter is more efficient because the Kalman equations are iterative in nature.

The learning mechanism of the Kalman filter can be simulated when we consider a situation, where the target vehicle hard-brakes suddenly resulting in a rapid reduction is relative distance and relative velocity. Consider that the host vehicle and target vehicle are initially in conditions described above for the first five measurements. Then the target vehicle hard brakes resulting in its relative velocity reducing to -20 mps with respect to the host vehicle. The randomness associated with the velocity of the target vehicle can be considered to increase by 4 times during hard braking and reduces down constantly. This sudden deceleration continues from fifth measurement to twentieth measurement and afterwards the vehicle starts moving in the uniform velocity and acceleration. The working of Kalman filter is simulated and results are shown in Figure 11.

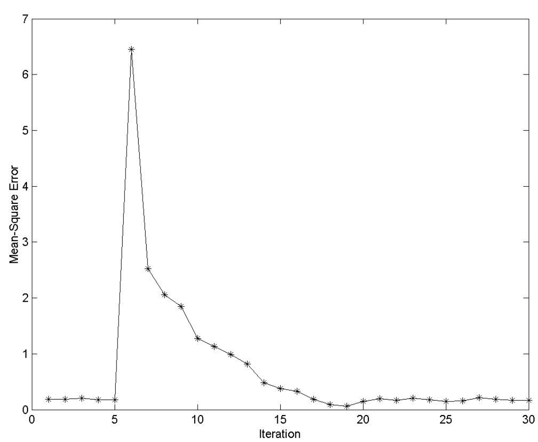

Figure 11. Performance of Kalman filter during hard braking of target vehicle

It can be observed that the mean-square error shoots up at the sixth measurement and slowly reduces down. At the twentieth measurement, the system stabilizes as the braking stops. This curve of reducing mean-square error is partly attributed to the learning nature of the Kalman filter and partly to the assumed constant reduction in randomness. The reduction in randomness during the period of hard brake of the target vehicle is a very practical assumption, as the driver of the host vehicle becomes alert about the braking as the time goes. Thus Kalman filter suits better as the prediction algorithm for the FCA system.

In practice, the performance of the predictor can be increased and its learning curve can be steepened drastically by introducing non-linearity through an extended Kalman filter (EKF)[24]. Most of the GPS systems for tracking the delivery trucks are known to be using EKF for prediction. The same non-linear equation can be used in the EKF of the DBF based FCA system. Also as said earlier, the introduction of EKF can improve the chance of converting the FCA system into a full-pledged adaptive cruise control system, since it would also require the same kind of prediction and velocity variations.

VII. Summary

This paper focused on designing a new architecture for vehicle safety that avoids forward collision by using the digital beamforming technique. The vitality of digital beamforming relies in the accuracy with which the highly directed array of sensors can operate in detecting and tracking the possible hurdle. When compared with the prevailing GPS based systems, its greatest advantage is its simplicity, applicability and ability to indigenously perform all calculations with in the vehicle's computer. During the analysis and simulation of DBF receiver to form directed beam in the desired angle, spectrum estimation algorithms to identify the direction of the hurdle and its other characteristics, and linear prediction algorithms for tracking, a lot of different approaches are compared for their performance and efficiency and a few recommendations are made. Those recommendations are summarized here.

For the DBF receiver design, more the number of array elements, more directed is the main lobe. The increase in the number of array elements also reduces the strength of sidelobes. A linear array of 16 elements can form a highly directed mainlobe to cater our need. Planer array may not be required, since the vertical movement of the vehicles on the road are unimportant and negligible. Barker sequence can be used for transmission. Direct digital synthesis can be used for generated signal at intermediate frequency. DDS can be implemented by using commercial DDS synthesizers, since its FPGA implementation is found to take up a lot of memory. The main assumption made in this design is that the signals are coming from far-field, which is reasonable. For signal estimation, eigen-based algorithm are the best in terms of noise performance and frequency resolution. But they are computationally expensive. So parametric methods are suited for the FCA system. The simulation results reveal that out of the parametric methods, unconstrained least-square method has a better frequency resolution and noise performance. The underlying assumption for suggesting parametric methods is that the received signal has positive SNR. This assumption is also valid in a non-military environment. In reality, unconstrained least-squares method can be used along with non-parametric windowing systems like Blackman-Tukey, there by making it an ARMA system. The choice of the windowing system itself is beyond the scope of this paper and it requires a better understanding of the actual operating environment. Out of the two linear prediction algorithms considered in this paper, Kalman filter is found to be performing better than Wiener filter in terms of computational efficiency and prediction accuracy. The measurement noise covariance of the system can be identified only after the entire system is implemented. More accurate the measurement noise covariance is, faster the convergence will be. For the FCA system, an EKF is preferred introducing non-linearity as per the study of operating environment. The non-linear function can be obtained by suitably modifying the non-linear function used the EKF of GPS based ground vehicle tracking systems. Along with EKF, adaptive filtering techniques like RLS algorithms can be used to improve the tracking.

In a digital beamforming system, the entire process from steering the beam, to identifying and tracking the target is done in digital domain. So any signal processor or microprocessor can be used to implement this system. However it is highly recommended to use multi-core digital signal processors that are specially designed for RADAR operations. The reason is that most of the recommended algorithms involve highly parallel mathematical operations which a multi-core DSP can handle better. An alternative is to use an ASIC or FPGA based system to implement the entire functionality. FPGA based implementations of Levinson-Durbin algorithm and Kalman filtering algorithm are found to be very efficient. Other than those approaches explained in this paper, there are many other types of beamformers and beamspace processors. The determination of the best approach is highly determinant on the environmental parameters. However the basic architecture of the digital beamformer does not change heavily among different methods.

REFERENCES

[1] J.R.Treat, N.S.Tumbas, S.T.McDonald, D.Shinar, R.D.Hume, R.E.Mayer, R.L.Stasifer, and N.J.Catellan, "Tri-level study of the causes of traffic accidents: Final report volume 1: Causal factor tabulations and assessments,institute for research in public safety," Indiana University, Tech. Rep., 1979.

[2] "Traffic collision statistics."

[3] C. Jordan and K. Balman, *Electromagnetic Waves and Radiating Systems*, 2nd ed. Prentice Hall, Inc., 1968.

[4] A. Di, "Multiple source location-a matrix decomposition approach," *IEEE Trans. on Acoustics, Speech and Signal Processing*, vol. ASSP-33, pp. 1086–1091, Oct. 1985.

[5] Y.Orino, M.K.Kurusowa, and T.Katagiri, "Direct-digital synthesis using delta-sigma modulated signals," *IEEE International Symposium on Circuits and Systems*, vol. 2, pp. 1706–1709, May 2005.

[6] T. Magazine, Ed., *On the Investigation of Hidden Periodicities with Application to a supposed twenty-six-day period of Meteorological Phenomena*, vol. 3, March 1898.

[7] J.G.Proakis and D.G.Manolakis, *Digital Signal Processing: Principles, Algorithms, and Applications.*, 3rd ed. Prentice Hall, Inc., 2000.

[8] N. Levinson, "The wiener rms error criterion in filter design and prediction," *J.Math.Phys.*, vol. 25, pp. 261–278, 1947.

[9] J. Durbin, "The fitting of time series models." *Rev. Inst. Int. Stat.*, vol. 28, pp. 233–243, 1960.

[10] L. M. Lee and H.-C. Wang, "An extended levinson-durbin algorithm for the analysis of noisyautoregressive process," *IEEE Signal Processing Letters, INSPEC-5178274*, vol. 3, pp. 13–15, Jan 1996.

[11] A.H.Nuttall, "Spectral analysis of a univariate process with bad data points, via maximum entropy and linear predictive techniques," NUSC, Tech. Rep., Mar 1976.

[12] S.R.Marple, "A new autoregressive spectrum analysis algorithm," *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. ASSP-28, pp. 304–314, Aug 1980.

[13] J.Rissanen, "A universal prior for the integers and estimation by minimum description length," *Ann. Stat.*, vol. 11, pp. 417–431, 1983.

[14] E.Parzen, "Some recent advances in time series modeling," *IEEE Trans. Automatic Control*, vol. AC-19, pp. 723–730, Dec 1974.

[15] R.D.Schmidt, "A signal subspace approach to multiple emitter location and spectral estimation," Ph.D. dissertation, Dept. of Electrical Engineering, Stanford University, Nov 1981.

[16] ——, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antenna and Propagation*, vol. AP-34, pp. 276–280, Mar 1986.

[17] M.Wax and T.Kailath, "Detection of signals by information theoritic criteria," *IEEE Trans. Acoustics, Speech, and Signal Processing*, vol. ASSP-32, pp. 387–392, Apr 1985.

[18] N.Wiener, "Extrapolation, interpolation and smoothing of stationary time series with engineering applications," 1949.

[19] V.C.Kelma and A.J.Laub, "The singular value decomposition: its computation and some applications," *IEEE Trans. on Automatic Control*, vol. AC-25, pp. 164–176, 1980.

[20] H. Choudhary, "Wiener filter solution by signular value decomposition (svd)," electrical and Computer Engineering Dept., University of Chicago, Illinois.

[21] R.E.Kalman, "A new approach to linear filtering and prediction problems," *Transaction of the ASME - Journal of Basic Engineering*, pp. 35–45, Mar 1960.

[22] R.G.Brown and P.Y.C.Hwang, *Introduction to Random Signals and Applied Kalman Filtering*, 2nd ed. John Wiley & Sons, Inc., 1992.

[23] O.L.R.Jacobs, *Introduction to Control Theory*, 2nd ed. Oxford University Press, 1993.

[24] S.J.Julier and J.K.Uhlmann, "A new extension of the kalman filter to nonlinear systems," *Int. Symp. Aerospace/Defense Sensing, Simul. and Controls*, 1997.

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